Year 10 – Higher Knowledge Organiser HT1

Indices:

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$x^0 = 1$

$(x^a)^b = x^{ab}$

- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
- $a^{-m} = \frac{1}{a^m}$

Product rule for counting:

m x n

Where m is the number of ways to do one task and for each of these, there are n ways of doing another task

Surds:

$$\sqrt{a} \times \sqrt{a} = a$$

Multiply surds: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

Divide surds: $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

Adding/subtracting surds:

$$a\sqrt{c} \pm b\sqrt{c} = a \pm b\sqrt{c}$$

Rationalise denominator:

$$\frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

$$\frac{a}{b+\sqrt{c}} \times \frac{b-\sqrt{c}}{b-\sqrt{c}} = \frac{ab-\sqrt{c}}{b^2-c}$$

Simplify:

$$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

Find a square number that goes into it.

Bounds:

How to find the upper and lower bound?

1. Half the degree of accuracy specified
2. Add to get the upper bound.
3. Subtract to get lower bound.

Find the lower and upper bound of 5.7 to 1 decimal place.

1. Half the degree of accuracy = 0.1 ÷ 2 = 0.05.
2. Upper bound = 5.7 + 0.05 = 5.75.
3. Lower bound = 5.7 – 0.05 = 5.65.

Recurring decimals:

$$0.\dot{5} = 0.55555\ldots..$$

$$0.\dot{5}\dot{7} = 0.5757575\ldots.$$

$$0.4\dot{8} = 0.4888888\ldots.$$

Example:

$$x = 0.5\dot{4}$$

$$10x = 5.4444444\ldots\ldots.$$

$$100x = 54.4444444\ldots\ldots.$$

$$90x = 49$$

$$x = \frac{49}{90}$$

Year 10 – Higher Knowledge Organiser HT2

Factorise- put into brackets

Difference of 2 squares:

$$a^2 - b^2 = (a+b)(a-b)$$

Complete the square:

$$x^2 + bx + c$$

$$(x + \frac{b}{2})^2 - \left(\frac{b}{2}\right)^2 + c$$

Expand- multiply out brackets

Arithmetic/ Geometric sequences

Arithmetic Sequences change by a common difference. This is found by addition or subtraction between terms

Geometric Sequences change by a common ratio. This is found my multiplication/ division between terms.

Term to term rule – how you get from one term (number in the sequence) to the next term

Position to term rule – take the rule and substitute in a position to find a term. E.g. Multiply the position number by 3 and then add 2.

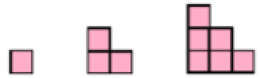
Other sequences

Fibonacci Sequence

1, 1, 2, 3, 5, 8 …

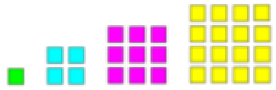
Each term is the sum of the previous two terms

Triangular Numbers – look at the formation



1, 3, 6, 10, 15 …

Square Numbers – look at the formation



1, 4, 9, 16…

Sequences are the repetition of a patten

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For a general equation:

$$ax^2 + bx + c = 0$$

Arithmetic nth term:

$$a_n = a + (n-1)d$$

where a_n = n^{th} term of the sequence
a = first term of the sequence
d = common difference

Geometric nth term:

$$a_n = ar^{n-1}$$

where a_n = n^{th} term of the sequence
a = first term of the sequence
r = common ratio

Quadratic nth term:

$a = \dfrac{\text{Second difference}}{2}$

b = The difference of each (Term - an^2)

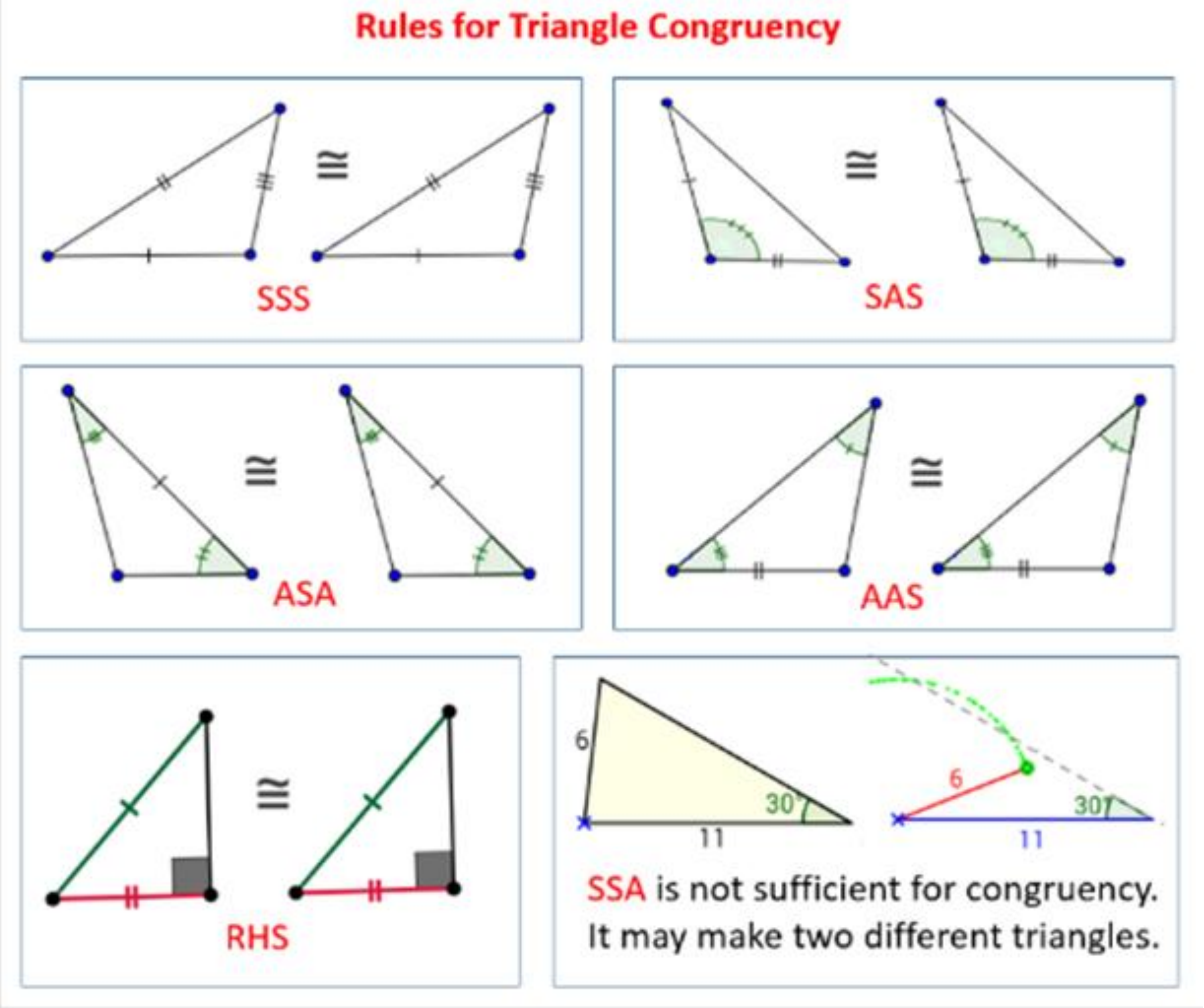
1^{st} term = $a + b + c$

Once you have found a, b and c you can slot these figures in the following formula.

$$an^2 + bn + c$$

Year 10 Higher – Half Term 3 Maths Knowledge Organiser

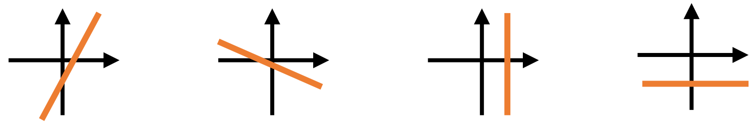
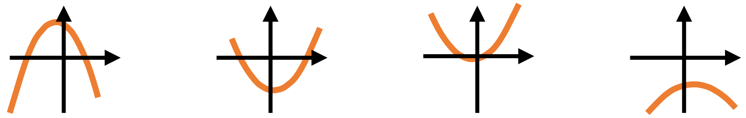
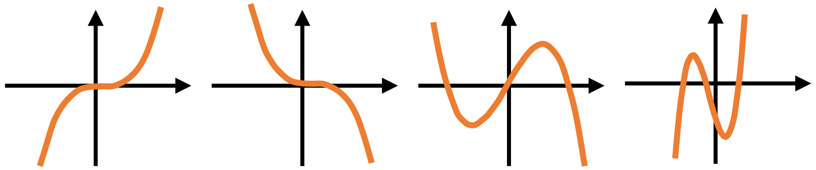
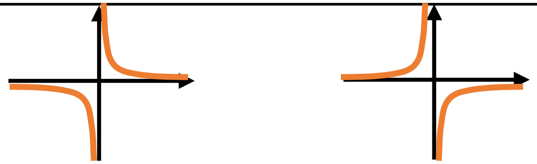
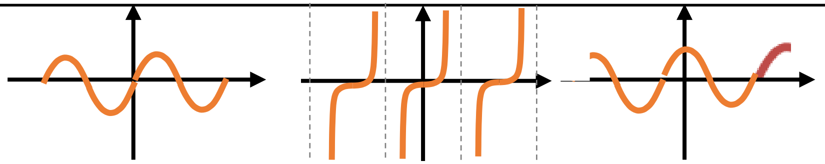
Proportion	Formula	
y is directly proportional to x	$y \propto x$	$y = kx$
y is proportional to x squared	$y \propto x^2$	$y = kx^2$
y is inversely proportional to x	$y \propto \frac{1}{x}$	$y = \frac{k}{x}$
y is inversely proportional to the square root of x	$y \propto \frac{1}{\sqrt{x}}$	$y = \frac{k}{\sqrt{x}}$



Similar shapes: Two shapes are similar if the angles are the same size or the corresponding sides are in the same ratio.

Length Scale factor	x
Area Scale factor	x^2
Volume Scale factor	x^3

Year 10 Higher – Half Term 4 Maths

Type	Equation types	Key features	Possible shapes
Linear	$y = mx + c$	▪ straight line shape ▪ m is the gradient ▪ c is the y-intercept	
Quadratic	$y = ax^2 + bx + c$	▪ u shape if **a** is +ve ▪ n shape if **a** is -ve ▪ can have up to 2 roots	
Cubic	$y = x^3$ $y = ax^3 + bx^2 + cx + d$	▪ goes up first if **a** is +ve ▪ goes down first if **a** is –ve ▪ can have up to 3 roots ▪ can have up to 2 turning points	
Reciprocal	$y = \frac{a}{x}$	▪ has 2 asymptotes has curves in the … ▪ 1st and 3rd quadrant if a is +ve ▪ 2nd and 4th quadrant if a is -ve	
Circular	$x^2 + y^2 = r^2$	▪ has a centre at (0, 0) ▪ has a radius of **r** units	
Exponential	$y = a^x$	▪ increasing when x is +ve ▪ decreasing when x is -ve ▪ passes through (0 , 1) ▪ asymptote at the x axis	
Trig	$y = \sin x$ $y = \cos x$ $y = \tan x$	▪ sin starts at (0, 0) ▪ cos starts at (0, 1) ▪ both repeat every 360° ▪ tan has asymptotes	

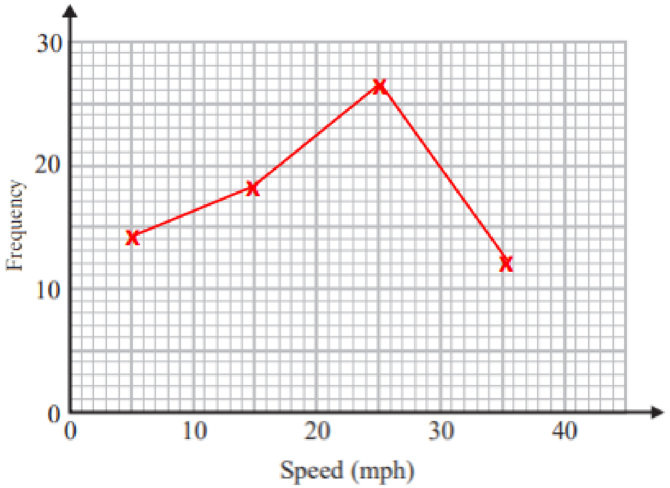
Year 10 Higher – Half Term 5 Maths

Drawing Frequency Polygons

This table gives information about the speeds of 70 cars.

Speed (s mph)	Frequency (f)	Midpoint
$0 < L \leq 10$	14	5
$10 < L \leq 20$	18	15
$20 < L \leq 30$	26	25
$30 < L \leq 40$	12	35

a) Draw a frequency polygon for this information.



Step 1 – Find the midpoint of each class interval

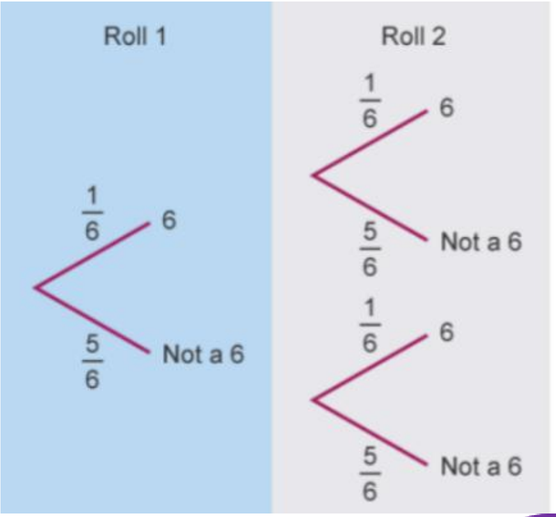
Step 2 – Label your axes and choose an appropriate scale

Step 3 – Plot each point at the midpoint for that interval

Step 4 – Connect each point with a straight line

Do not extend the line beyond the points you have

Probability Tree diagrams are a way of showing combinations of two or more events



6, 6 = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

6, not a 6 = $\frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$

Not a 6, 6 = $\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

Not a 6, not a 6 = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

Scatter Graphs

Positive correlation
As one variable increases so does the other variable.

Negative correlation
As one variable increases the other variable decreases.

No correlation
There is no relationship between the two variables.

Key Terms:

Discrete data: countable data that can be categorised e.g. *Shoe size, eye colour*

Continuous data: data that is measured and can take any value e.g. *Height, time, temperature*

Qualitative data: text-based data that describes something e.g. *colours, race*

Quantitative data: numerical data e.g. *age, height, temperature*

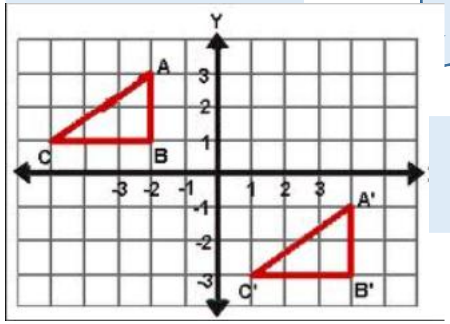
Frequency: the number of occurrences of an event

Extrapolate: to predict values from outside the range of data

Year 10 Higher – Half Term 6 Maths KO

Transformations

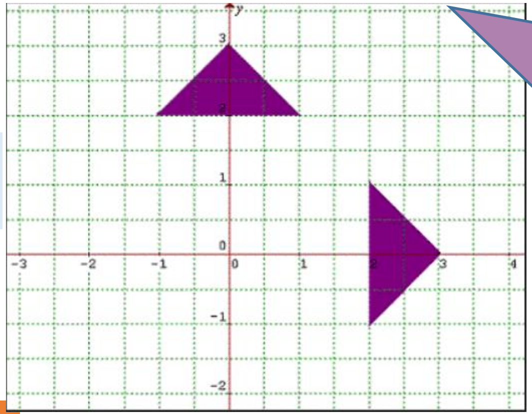
Translations



Describe with a vector

$\begin{pmatrix}3\\2\end{pmatrix}$ Squares right/left
Squares up/down

Rotation

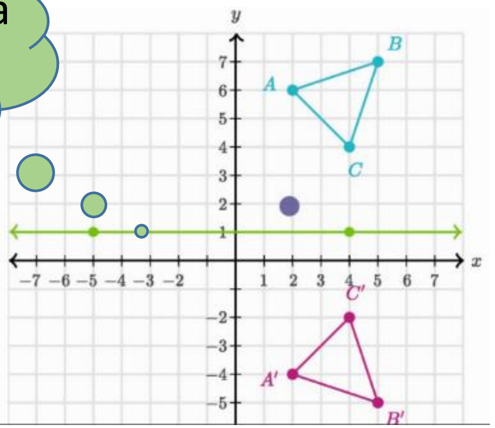


To describe a rotation you need:

- The angle of rotation
- The direction
- The coordinates of the centre

Reflection

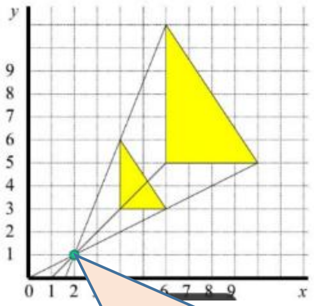
Describe with a line of symmetry

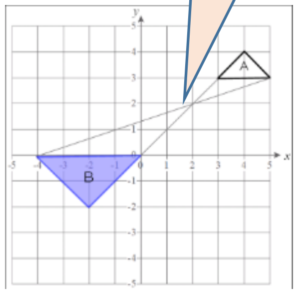


Reflection in the line y=1

Use tracing paper for reflection and rotation

Enlargement



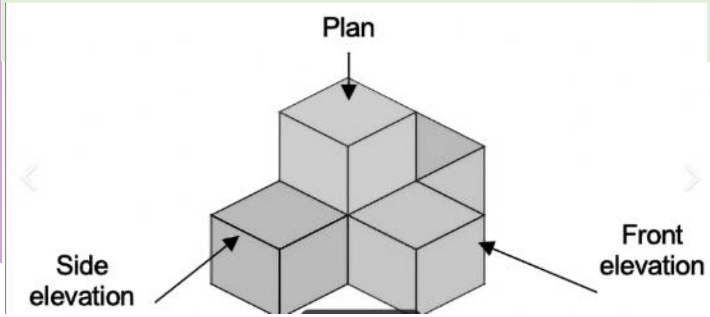


Negative enlargement By SF -2

To describe an enlargement you need:

- Scale factor
- Centre of enlargement

Plans and elevations:



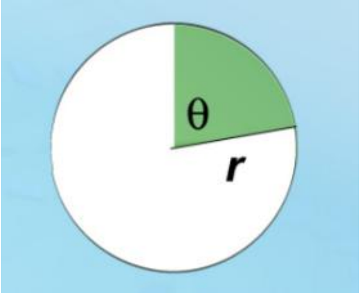
Circles:

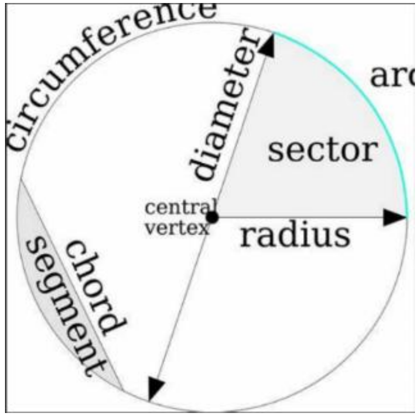
Area of a circle $= \pi r^2$

Circumference of a circle = πd

Area of a sector of a circle $= \frac{\theta}{360}\pi r^2$

Arc length of a sector of a circle$=\frac{\theta}{360}\pi d$





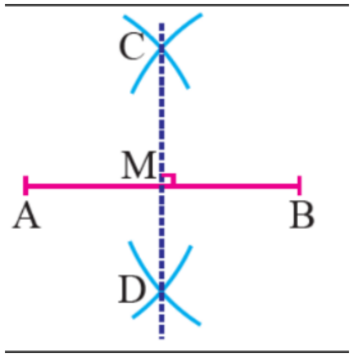
Year 10 Higher – Half Term 6 Maths KO

Constructions:

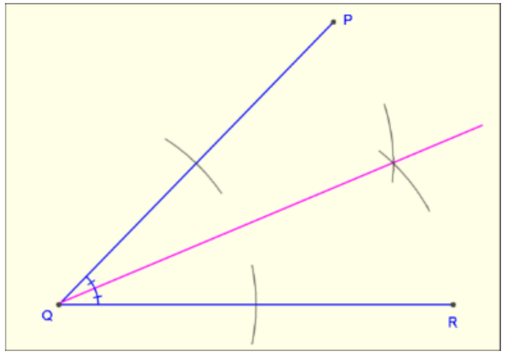
Perpendicular bisector – cuts a line in half

Bisector of an angle- cuts an angle in half

Always leave construction marks

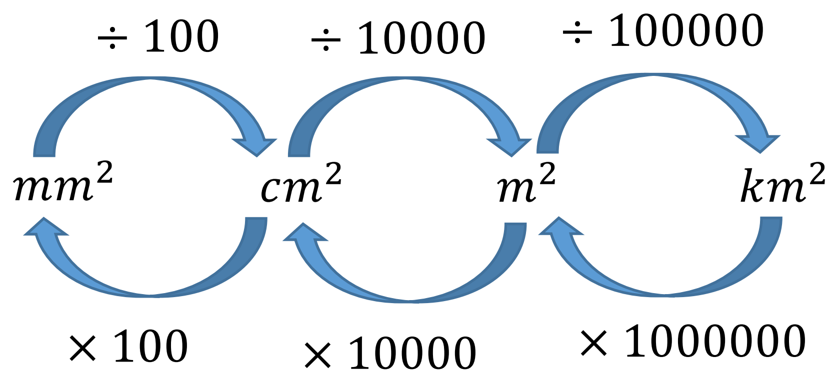


Perpendicular bisector



Bisect an angle

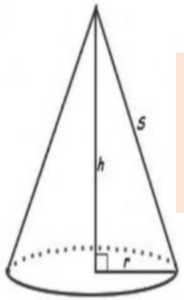
Convert between metric area:



Surface area and volume:

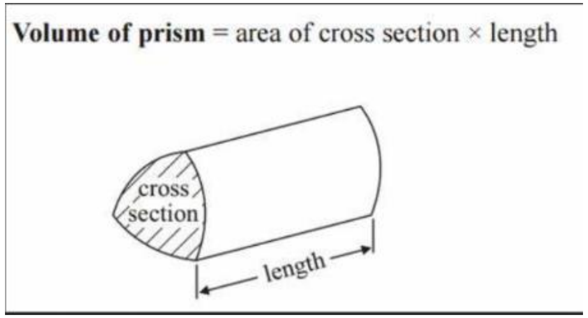
Surface area – the **total area of the surface of a three-dimensional object**

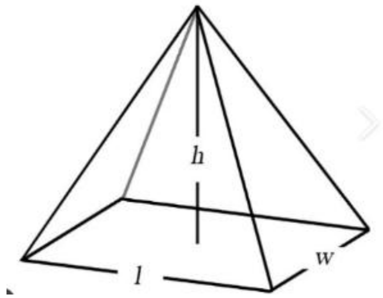
Volume- **shows the amount of three-dimensional space occupied by an object**



Volume=$\frac{1}{3}\pi r^2 h$

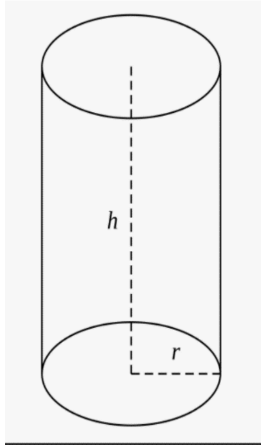
Surface area= $\pi r^2 + \pi r s$





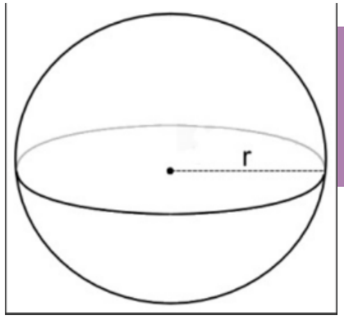
Volume=$\frac{1}{3} lwh$

Surface area= area of base + 4 x area of triangle



Volume= $\pi r^2 h$

Surface area=$2\pi r^2 + \pi 2r$



Volume= $\frac{4}{3}\pi r^3$

Surface area= $4\pi r^2$